Ordinary Integral of Vectors

Semester IV – MJC Physics

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1. Introduction

In physics, many quantities such as **displacement, velocity, force, electric field**, etc. are **vector quantities**.
When a vector varies with respect to a scalar variable (like time or distance), we often need to **integrate vectors**. This process is known as **ordinary integration of vectors**.

Ordinary vector integration is mainly used in:

- Kinematics

- Work and energy

- Electromagnetism

- Classical mechanics

2. Definition of Vector Integration

If

$$\vec{A}(x) = A_x(x)\hat{i} + A_y(x)\hat{j} + A_z(x)\hat{k}$$

then the **ordinary integral** of the vector is defined as:

$$\int \vec{A}(x)\,dx = \int A_x(x)\,dx\,\hat{i} + \int A_y(x)\,dx\,\hat{j} + \int A_z(x)\,dx\,\hat{k}$$

3. Ordinary Integral of a Constant Vector

Let

$$\vec{A} = a\hat{i} + b\hat{j} + \cdots$$

Then,

$$\int \vec{A}\, dx = (ax)\hat{i} + (bx)\hat{j} + (cx)\hat{k} + \vec{C}$$

where $\vec{C}$ is the **constant vector of integration**.

4. Integral of a Vector Function of Time

If velocity vector is given as:

$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z$$

Then displacement vector:

$$\vec{r}(t) = \int \vec{v}(t)\, dt$$

This is widely used in **particle motion**.

5. Properties of Vector Integration

(i) Linearity

$$\int (\vec{A} + \vec{B})\, dx = \int \vec{A}\, dx + \int \bar{E}$$

(ii) Constant Multiplication

$$\int k\vec{A}\, dx = k \int \vec{A}$$
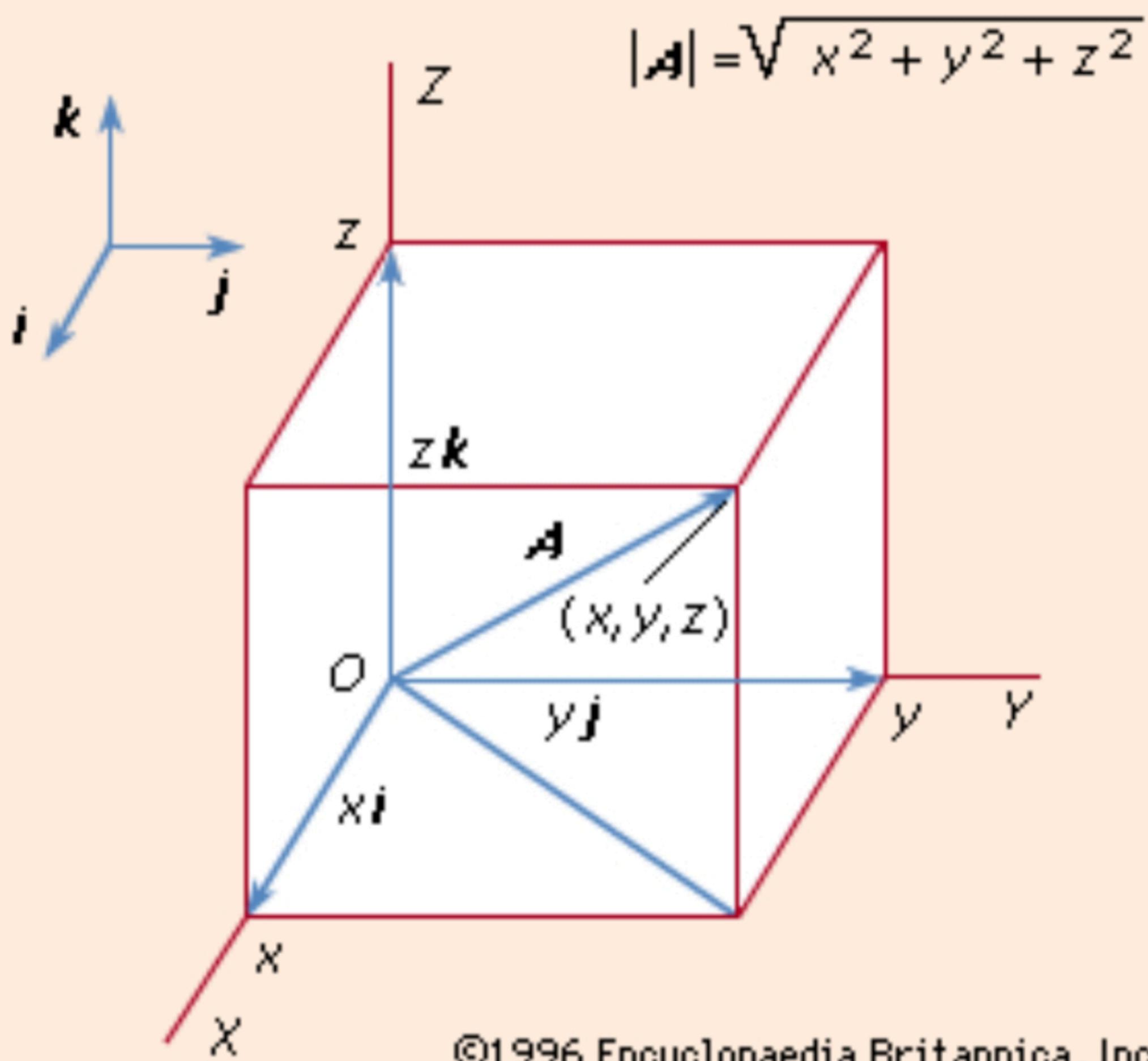
(iii) Zero Vector

$$\int \vec{0}\, dx = ($$

6. Definite Integral of a Vector

If a vector depends on x and limits are from a to b:

$$\int_a^b \vec{A}(x)\, dx = \left[\int_a^b A_x\, dx\right] \hat{i} + \left[\int_a^b A_y\, dx\right] \hat{j} + \left[\int_a^b A_z\, dx\right] \hat{k}$$

7. Physical Interpretation

- Integral of **velocity vector** → displacement

- Integral of **force vector w.r.t. distance** → work

- Integral of **electric field** → electric potential difference

$$|A| = \sqrt{x^2 + y^2 + z^2}$$

k

j

i

Z

z

k

zk

A

(x, y, z)

O
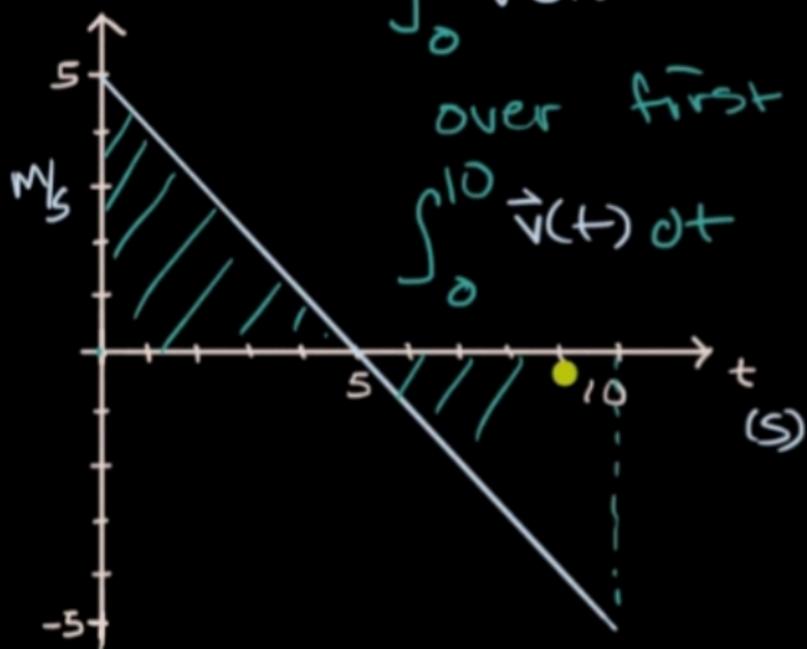
yj

y   Y

xi

x

X

Displacement = change in position

$\vec{v}(t) = 5 - t$

Displacement over first 5 seconds

$$\int_0^5 \vec{v}(t)\, dt = 12.5 \text{ m}$$

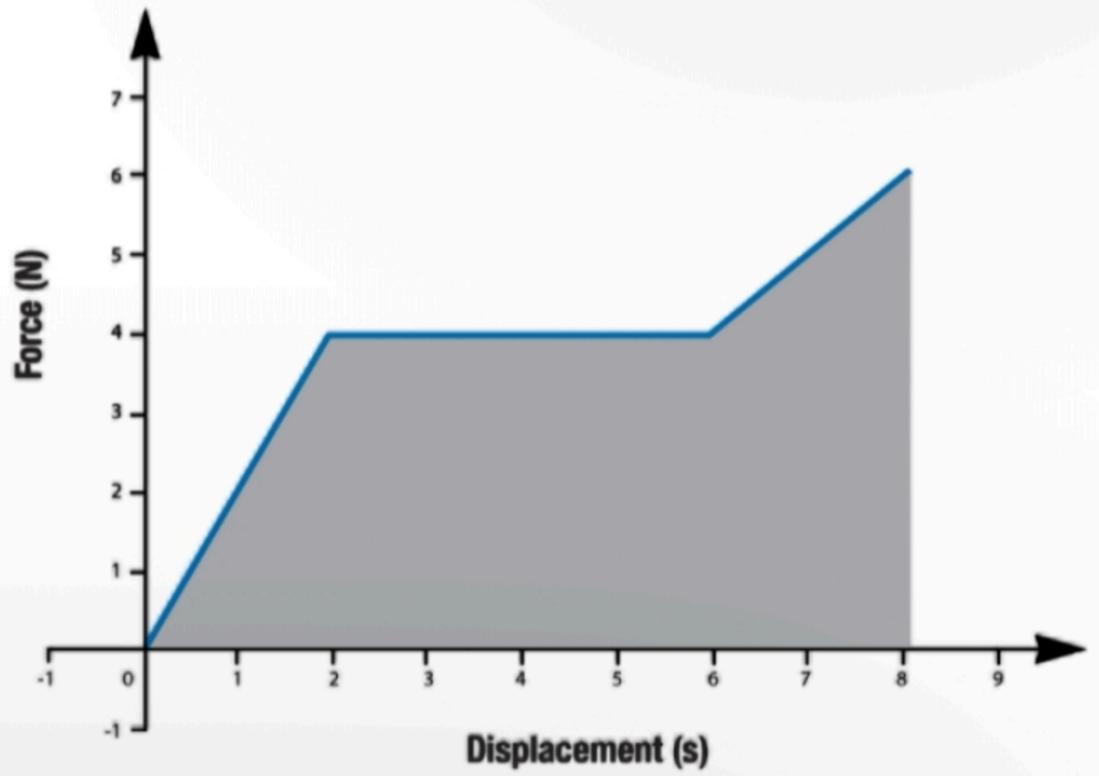over first 10 seconds

$$\int_0^{10} \vec{v}(t)\, dt$$

Distance traveled = total length of path

8. Solved Example 1

Given:

$$\vec{A}(x) = (2x)\hat{i} + (3x^2)\hat{j} + (4)\hat{k}$$

Solution:

$$\int \vec{A}\, dx = \int 2x\, dx\, \hat{i} + \int 3x^2\, dx\, \hat{j} + \int 4\, dx\, \hat{k}$$

$$= x^2\hat{i} + x^3\hat{j} + 4x\hat{k} + \vec{C}$$

9. Solved Example 2 (Physics Based)

Given velocity:

$$\vec{v}(t) = (3t)\hat{i} + (2t^2)\hat{j}$$

Find displacement.

Solution:

$$\vec{r}(t) = \int \vec{v}(t)\, dt$$

$$= \left(\frac{3t^2}{2}\right)\hat{i} + \left(\frac{2t^3}{3}\right)\hat{j} + \vec{C}$$

10. Important Exam Points

✔ Vector integration is always **component-wise**

✔ Constant of integration is a **vector**

✔ Used mainly in **mechanics and electromagnetism**

✔ Very common **2–5 marks question** in Semester IV

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11. Conclusion

Ordinary integration of vectors is a fundamental mathematical tool in physics. It simplifies the study of motion, force, and field-related quantities by reducing vector problems into manageable scalar integrations.